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Determination of joint functional axes from noisy marker data using the finite helical axis

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Abstract

When video-based motion analysis systems are used to measure segmental kinematics, the finite helical displacement computed between two adjacent body segments in two successive positions i, $i+1$ is often used to approximate the instantaneous joint movement. The measured trajectories of the external markers glued on the skin are very perturbed compared to the real displacement of the bony structure, and the inaccuracy in the measurement leads to stochastic errors in the position and direction of the finite helical axis of motion (FHA). As the errors associated with the FHA estimates are inversely proportional to the rotaion magnitude (Woltring, H.J., Huiskes, R., de Lange, A., 1983. Measurement error influence on helical axis accuracy in the description of 3D finite joint movement in biomechanics. In: Woo, S.L., Mates, R.E. (Eds.), Biomechanics symposium AMD 56 (FED 1), New York ASME, pp. 19–22), it is illusive to expect to assess the helical displacement between two neighbouring positions, and so to describe the joint evolution using FHA theory in such a context.

A quantification of the errors on the FHA parameters computed between two successive positions i, $i+1$ is proposed in this paper, using a numerical simulation of the knee joint evolution during gait. This case has been chosen because previous studies (Cappozzo, A., Catani, F., Leardini, A., 1993. Skin movement artifacts in human movement photogrammetry. Proceedings of the 14th Congress of the International Society of Biomechanics. Paris, France,

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pp. 238–239) have experimentally described skin and soft tissues perturbations. The results obtained from this simulation lead to two conclusions: first, they confirm the relative FHA cannot be used to represent accurately the joint kinematics during a given movement; and second, they allow a prediction of the minimum joint displacement required in order to have a reliable determination of the helical axis.

The aim of this paper is not to present a new calculation method for the FHA, but to propose an alternative use of the FHA. It is generally assumed to describe a joint displacement using a sequence of rotations about three successive axes. In this case, the difficulty for clinical applications is to correctly locate these axes, in order that they coincide with the functional axes of the considered joint. If the FHA theory is used to determine the location and orientation of these functional axes from corresponding pure movement recording, then the results can be very accurate provided that the measured displacement between the two finite positions i and j be sufficient with respect to the perturbing noise. One consequence of this remark is that the rotation axis of the considered, joint may remain stable in the range of motion between these positions.

An example of this altenative use of the FHA is displayed in this paper which concerns the determination of the flexion/extension axis of the elbow joint. The elbow joint has been chosen for two reasons: first, it deals with a stable joint rotation axis and second, experimental data were available on a fleshless upper-limb on which the flexion/extension axis of the elbow was marked by an anatomy specialist.

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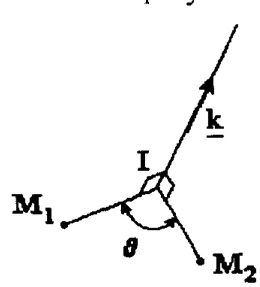
1. Introduction

In a mechanical context, the finite displacement of a rigid body between two positions can be modelled using the finite helical axis (FHA) theory. It corresponds to a rotation of magnitude θ plus a translation of magnitude about the same axis, where position in space is defined by one point I and of which orientation is given by a unit vector $\mathbf{k}$.

In the case of a finite rotation without translation ($t = 0$), the position M_2 of any point fixed to the rigid body after rotation is deduced from its initial position M_1 by the following relationship written by Rodrigues, 1840 as



$$M_1 M_2 = 2\,\mathbf{k}\ \tan(\theta/2) \times \left(\frac{IM_1 + IM_2}{2}\right). \tag{1}$$

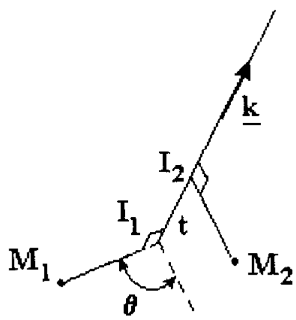
This relationship has been developed by Dimnet and Guingand (1984) to express the desired position M_2 as a function of known quantities

$$M_1 M_2 = \sin\theta\,(\mathbf{k} \times IM_1) - (1 - \cos\theta)\,IM_1,$$

or in a different way

$$IM_2 = \sin\theta(\mathbf{k} \times IM_1) + \cos\theta\,IM_1.$$

In the case of an helical displacement ($t \neq 0$), the Rodrigues formula becomes



$$M_1 M_2 = 2\,\mathbf{k}\ \tan(\theta/2) \times \left(\frac{I_1M_1 + I_2M_2}{2}\right) + I_1I_2. \tag{2}$$

From a biomechanical point of view, the problem to be solved is inverse. The successive positions in space of at least three points fixed on each body segment are measured, and the objective is to obtain the characteristics of the joint displacements, i.e. the relative displacements between two adjacent bones.

The main problem is that the measurements often correspond to the spatial positions of external markers glued on the skin computed using a video-based motion analysis system. Actually, these marker trajectories are noisy compared to the real displacements of the underlying bony landmarks. Several researchers (Andriacchi, 1987; Angeloni et al., 1992; Lafortune et al., 1992) have shown that these perturbations are significant (e.g., relative displacements of about 2 cm between external markers fixed on the thigh segment and corresponding landmarks on the femur).

Different solutions are proposed to reduce the effect of these measurement errors. For example, Veldpaus et al. (1988) and Sodërkvist and Wedin (1993) determine the kinematics of the body segments from noisy marker trajectories by using least-square techniques. Chèze et al. (1995) perform a "solidification" of the body segments, which consists in substituting trajectories consistent with the rigid state assumed for the bones to the measured trajectories of the external markers. These kinds of methods allow the correction of the relative displacements between markers belonging to the same body segment, i.e. the non-rigidity of the segment due to the skin elasticity. Nevertheless, they are unable to correct the global displacements of the set of external markers with respect to the underlying bone, due to muscular and adipose tissues.

As a common rule, when one disposes of a sampled recording of the movement, the finite helical displacement computed between two successive positions i, $i+1$ is used to approximate the instantaneous helical axis (IHA) parameters. Actually, the IHA determination requires a good estimate of both the positions and velocities of the measured points, which are quite difficult to obtain. Nevertheless, if one uses noisy experimental data, the inaccuracy in the measurement leads to stochastic errors in the position and direction of the FHA. The error propagation formulas described by Woltring et al. (1983) and Spoor (1984) have pointed out that these errors are inversely proportional to the rotation magnitude, and this rotation magnitude is generally small between two successive positions i and $i+1$. In consequence, it becomes illusive to expect to obtain the characteristics of the helical displacement between two neighbouring positions.

A numerial simulation is proposed on the knee joint to quantify these errors within typical experimental conditions. It is shown that the determination of the FHA parameters is accurate enough, provided that the measured displacement be sufficient with respect to the perturbing noise. Curves are drawn as a function of the joint rotation amplitude in order to anticipate the accuracy obtained on each FHA parameter for a given data perturbation. From these results, an alternative use of the FHA concept is proposed. It consists in computing the FHA characteristics between two sufficiently separate positions of the joint performing a pure movement, in order to assess the corresponding functional axis. The interest of the FHA computation for a functional axis determination is illustrated using a clinical experiment on the flexion/extension axis of the elbow joint.

2. Method

2.1. Computation of the FHA characteristics

One of the methods often used to determine the characteristics of the helical displacement of a solid from the measurement of the successive positions of n points ($n \geq 3$) is that described by Spoor and Veldpaus (1980) and Veldpaus et al. (1988). It consists of determining the rotation matrix $\mathbf{R}$ and the translation vector $\mathbf{t}$ characterizing the relative finite displacement from noisy data using a least-square technique. Let X_i and Y_i be the coordinates, in the reference frame of the upper segment (supposed fixed in the relative displacement), of a given point M_i belonging to the lower segment in two distinct positions of the solid; the matter is to solve the following equation

$$\min \sum_{i=1}^{n} \|\mathbf{R}X_i + \mathbf{t} - Y_i\|^2.$$

Sodërkvist and Wedin (1993) propose to solve this equation by using an algorithm of Singular Value Decomposition (SVD) which improves the solution's stability.

From the matrix $\mathbf{R}$ and the vector $\mathbf{t}$, one can obtain easily the characteristics of the equivalent helical displacement. Actually, the relative rotation operator $\mathbf{R}$ can be written as follows:

$$\mathbf{R} = \begin{bmatrix} k_x^2 v\theta + c\theta & k_y k_x v\theta - k_z s\theta & k_z k_x v\theta + k_y s\theta \\ k_x k_y v\theta + k_z s\theta & k_y^2 v\theta + c\theta & k_z k_y v\theta - k_x s\theta \\ k_x k_z v\theta - k_y s\theta & k_y k_z v\theta + k_x s\theta & k_z^2 v\theta + c\theta \end{bmatrix} = [\alpha_{ij}],$$

where $s\theta = \sin(\theta), c\theta = \cos(\theta)$ and $v\theta = 1 - \cos(\theta)$.

From the coefficients of this operator, the rotation magnitude can be derived as follows

$$\theta = \cos^{-1}\left(\frac{\alpha_{11} + \alpha_{22} + \alpha_{33} - 1}{2}\right).$$

The direction of the FHA, i.e. the components of a unit vector in the upper segment reference frame (supposed fixed), are given by the relations

$$\mathbf{k} = \begin{bmatrix} \dfrac{\alpha_{32} - \alpha_{23}}{2\ \sin(\theta)} \\ \dfrac{\alpha_{13} - \alpha_{31}}{2\ \sin(\theta)} \\ \dfrac{\alpha_{21} - \alpha_{12}}{2\ \sin(\theta)} \end{bmatrix}.$$

The translation's magnitude is obtained through the scalar product: $t = \mathbf{t} \cdot \mathbf{k}$. The position in space of the FHA, i.e. the coordinates of one point I_1 belonging to the axis, is obtained using the Rodrigues formula Eq. (2) which can also be written as

$$I_1M_1 = -\frac{1}{2}M_1M_2 + \frac{1}{2}t\mathbf{k} - \frac{1}{2\ \tan(\theta/2)}(\mathbf{k} \times M_1M_2).$$

The question is to know under what conditions this kind of method, currently used to describe the joint displacement between two measured positions, gives accurate enough results. To answer this question, it is advisable to use a numerical simulation, best reproducing the trajectories of external markers glued on the skin, in order to quantify the error due to the noise on the bone displacements determination.

2.2. Numerical simulation

The quantification of the errors on the FHA parameters is proposed on a numerical simulation of the knee joint evolution during gait, because measurements of the data perturbations are available in the literature (Cappozzo et al., 1993).

To obtain realistic marker displacements, the reference movement is built using experimental trajectories. These trajectories are those of three markers glued on the thigh and of three markers on the shank, measured using a video-based motion analysis system (Motion Analysis Corp., Santa Rosa – CA) during a gait cycle. The subject (a 25 year old male) was walking at a natural speed (about 4.5 km/h) and the movement was recorded at a frequency of 60 Hz. From the spatial positions of the three markers fixed on each segment, the method previously described gives the characteristics of the helical displacement corresponding to the relative movement between the thigh and the shank (or knee joint movement). These characteristics are: the components of the unit vector $\mathbf{k}$ of the axis, the coordinates of one point I belonging to the FHA and the rotation θ and translation t magnitudes. These parameters are then filtered to obtain a reference movement smoothly repetitive. This filtering is realized using a low-pass Butterworth filter, of which cut-off frequency is chosen equal to 3 Hz in order to eliminate the rough variations due to measurement noises. The successive positions of the shank with respect to the thigh are then recomputed to provide a non-perturbed 3D "reference" movement of the knee joint. For this, the relative rotation operator $\mathbf{R}$ and the translation vector $\mathbf{t}$ are calculated for each position i from the filtered helical movement characteristics. Then, these kinematics are applied to an unperturbed triangle representing the marker configuration on the shank, the positions of the thigh remaining unchanged. The thigh segment is rigidified by applying the "solidification" procedure on the marker trajectories (Chèze et al., 1995). In this way, the reference trajectories correspond to a regular and quite realistic knee displacement, whose characteristics are exactly known.

To these reference trajectories, random noise representing the measurement errors due to perturbating displacements of external markers with respect to the corresponding bony landmarks are added. Since Cappozzo et al. (1993) found that skin-fixed markers move in a continuous rather than random fashion relative to their underlying anatomical landmarks, a continuous noise model of the form $A\sin(\omega t + \phi)$ is chosen where A is the amplitude of the noise, ω the frequency, t the simulated time and ϕ the phase angle. The parameters A, ω and ϕ are random numbers scaled to represent the motion artifacts anticipated during gait. The choice of a continuous perturbation is open to criticism for gait, where heel contacts induce impacts. Nevertheless, it is appropriate for movements undertaken to determine functional axes. Actually, in such a context, pure movements about each functional axis are performed slowly and regularly.

Since skin and soft tissue perturbations as large as 2 cm have been observed experimentally (Andriacchi, 1987; Cappozzo et al., 1993), each amplitude A is scaled to be between 0 and 1 cm (i.e. a 2 cm range). Similarly, since such perturbations, typically contain the same frequencies as those of the movement (Mann and Antonsson, 1983), each frequency ω is scaled to be between 0 and 25 rad s^{-1} (to cover the gait movement frequencies). Finally, to allow any phase relationship between the various perturbing sine functions, each phase angle ϕ is scaled to be between 0 and 2π rad (see Chèze et al., 1995).

Then the relative FHA characteristics are calculated from these perturbed marker trajectories, using the previously described method, first between successive positions i, $i + 1$ and second between any positions i and j. The errors measured on the FHA parameters are assumed representative of results obtained from experimental trajectories of markers glued on the skin.

3. Results

3.1. Quantification of the error on FHA parameters using a numerical simulation

Let us first show the stochastic parameters obtained when the FHA is computed as usual between two successive positions i, $i + 1$. Fig. 1(a) and (b) dis-

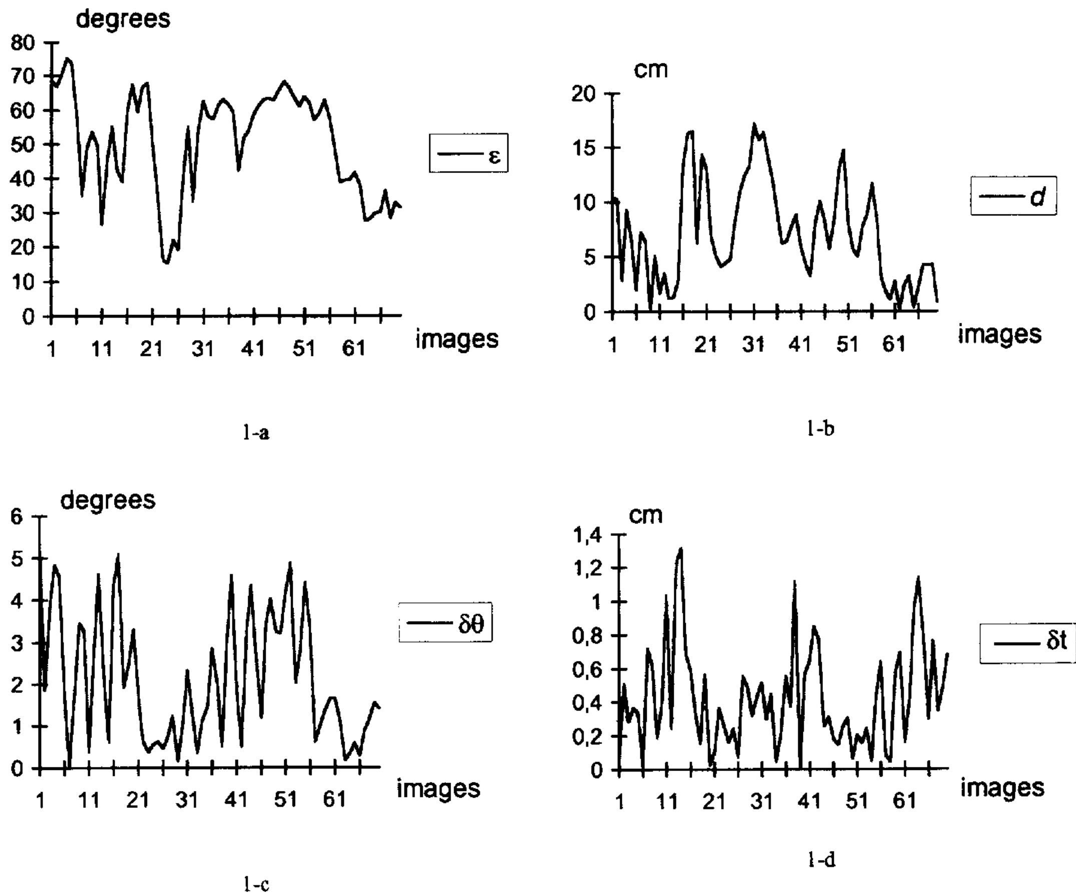


Fig. 1. Errors in the FHA characteristics, as a function of image numbers (sampling frequency: 60 Hz), corresponding to a simulation of experimental data obtained using external markers glued on the skin (maximum amplitude of noise = 2 cm). (a) Angle ε between the reference movement FHA and the FHA computed using noisy data. (b) Linear distance d between these two FHAs. (c) Error $\delta\theta$ in the rotation magnitude. (d) Error δt in the translation magnitude.

play respectively the angular error ε in the direction, and the linear distance d between the reference movement's axis and the axis computed from the noisy data.

Let $\mathbf{k}_r$ and $\mathbf{k}_p$ be the unit vectors of the FHA computed from the unperturbed data and from the perturbed trajectories for each position i, respectively. The angular error is computed by the relation: $\varepsilon = \cos^{-1}(\mathbf{k}_r \cdot \mathbf{k}_p)$. If I_r and I_p are the points belonging to the reference and the perturbed FHA respectively, and $\mathbf{n}$ a unit vector perpendicular to both vectors $\mathbf{k}_r$ and $\mathbf{k}_p$ obtained by $\mathbf{n} = \mathbf{k}_r \times \mathbf{k}_p$, the linear distance d is obtained by the relation: $d = |I_r I_p \cdot \mathbf{n}|$.

Fig. 1(c) and (d) show the errors on the rotation magnitude $\delta\theta$ and translation magnitude δt, calculated hy the absolute values of the differences: $|\theta_r - \theta_p|$ (respectively: $|t_r - t_p|$).

The main errors deal with the direction and position of the axis: the angular error ε varies from 15° to 75° with a mean value of about 50°. The axis position is not well assessed either, as the maximum distance d between the reference and the calculated axis is quite important (17 cm) and the mean value is about 7 cm. The errors on the movement variables $\delta\theta$ and δt are smaller, the maximum values are 5° for the rotation and 1.3 cm for the translation respectively.

These results show clearly that the location and direction of the FHA are not well assessed when the perturbations are of the same magnitude as the measured displacement. The idea is to define a mean helical axis between

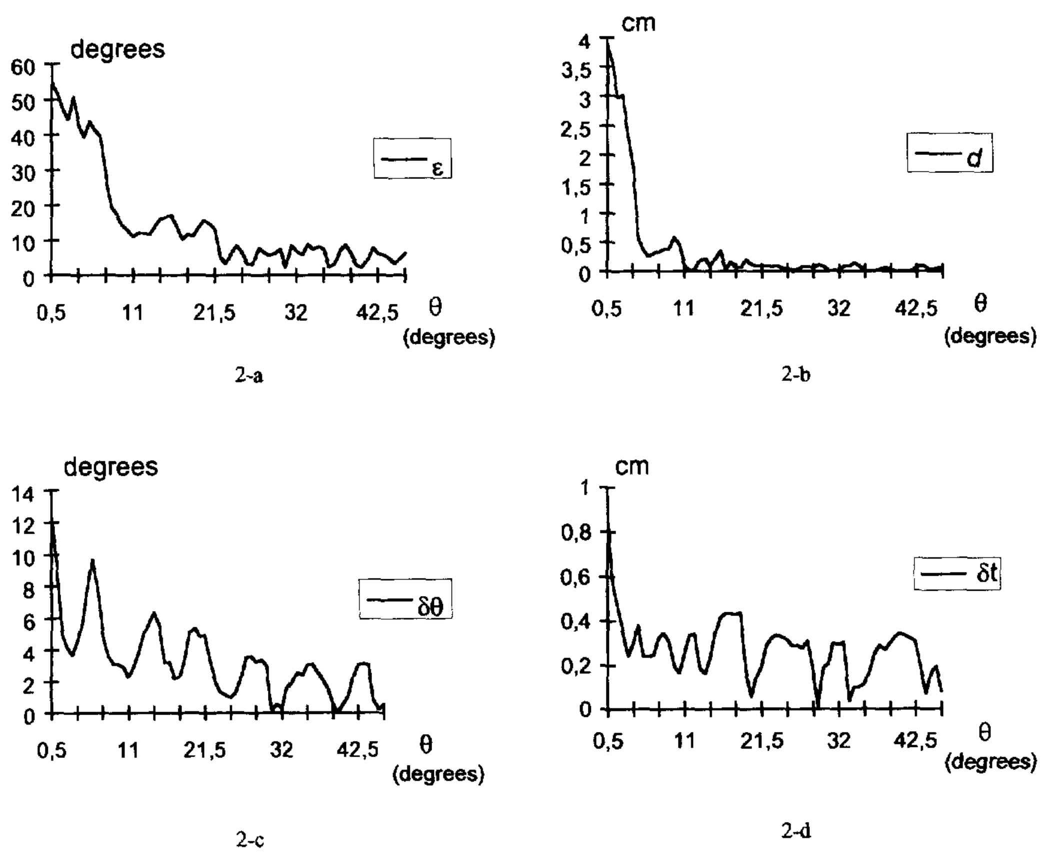


Fig. 2. Errors in the FHA characteristics, as a function of the rotation magnitude θ, corresponding to the same simulation (maximum amplitude of noise = 2 cm). (a) Angle ε between the reference movement FHA and the FHA computed using noisy data; (b) Linear distance d between these two FHAs; (c) Error $\delta\theta$ in the rotation magnitude; (d) Error δt in the translation magnitude.

two sufficiently distinct positions i and j. Fig. 2 displays the variation of the perturbations' effect on the different FHA parameters as a function of the rotation magnitude θ between the images i and j. As expected from the propagation formulas described by Woltring et al. (1983) and Spoor (1984), we can notice a significant decrease in the error on the axis direction (angle ε) and on the axis position (distance d) when the amplitude of the measured displacement between the two considered positions is increased. We can consider that the direction is obtained with a good accuracy ($\varepsilon \leq 10°$) when the variation between the two positions is above 22° (Fig. 2(a)). The error on the position in space also decreases rapidly with the rotation θ and becomes quite small ($d \leq 0.5$ cm) as soon as the rotation magnitude is more than 10° (Fig. 2(b)). As far as the magnitudes θ and t are concerned, the improvement of the results for a movement of great magnitude is less obvious. The error on the rotation magnitude $\delta\theta$ is about 4° for a variation greater than 22° (Fig. 2(c)). The error on the translation magnitude δt does not depend significatively on θ, except for very small values, but remains below 0.5 cm as soon as the rotation θ is more than about 2° (Fig. 2(d)).

These results correspond to experimental conditions for which the measured positions are those of external markers glued on the skin over the corresponding bony landmarks, i.e. to measurement errors of about 2 cm maximum. A second range of trials has been tested, choosing a maximum error amplitude of 0.5 cm (i.e. an amplitude A for the noise model scaled to be between 0 and 0.25 cm). This case corresponds to another kind of experimental data, where the anatomical points are directly digitalized on radiographs. Under these new conditions, we obtain similar variations of the errors on the various parameters, but the error magnitudes are definitely smaller (Fig. 3).

3.2. Determination of the functional joint axis from experimental data

From the previous results, the position and orientation of the FHA is well defined only if the relative displacement between the two considered positions i and j is sufficient with respect to the measurement noise (Figs. 2 and 3). So, this concept of finite helical axes does not allow a reliable description of the variation of the joint displacement from external markers glued on the skin. Actually, to follow the joint movement, two successive positions i and $i + 1$ have to be considered, and then the real displacement and the measurement noise are of about the same magnitudes. On the other hand, the FHA can be obtained with a good accuracy when it is computed between two sufficiently distinct distinct positions i and j. As a consequence, it can be considered as a

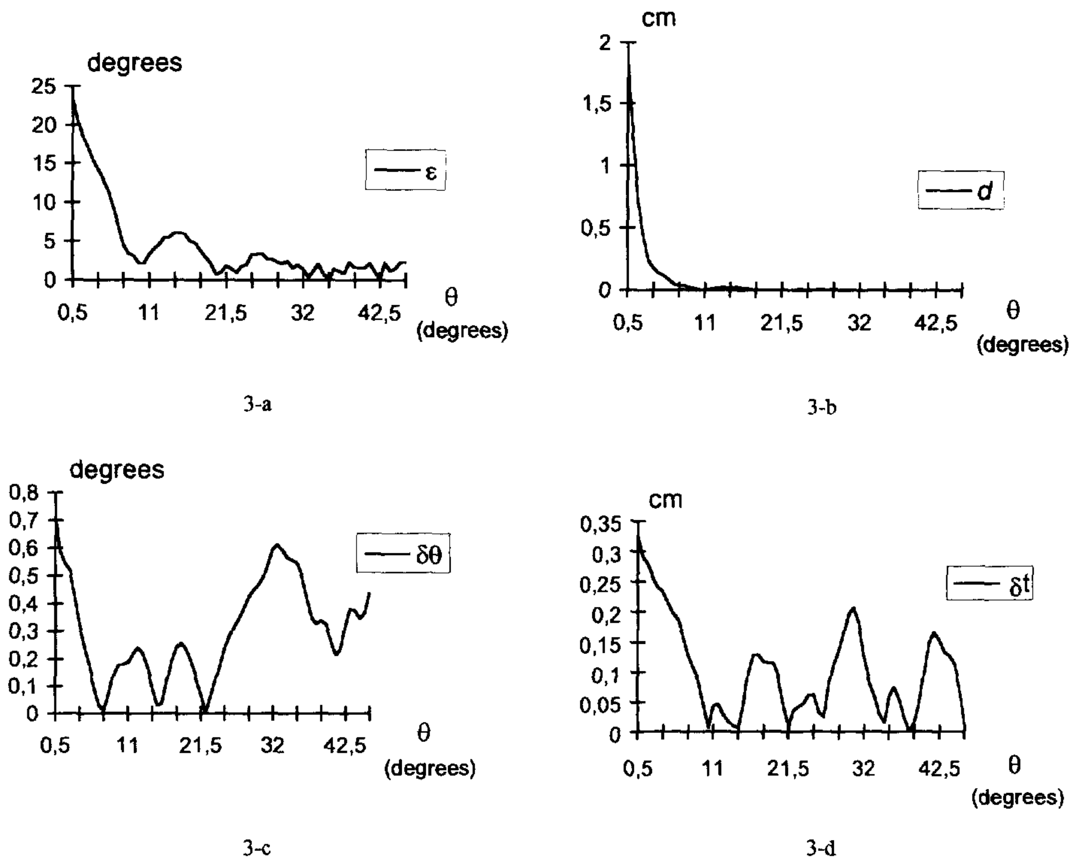


Fig. 3. Errors in the FHA characteristics, as a function of the rotation magnitude θ, corresponding to a simulation of experimental data obtained digitalizing anatomical landmarks on radiographs (maximum amplitude of noise = 0.5 cm). (a) Angle ε between the reference movement FHA and the FHA computed using noisy data; (b) Linear distance d between these two FHAs; (c) Error $\delta\theta$ in the rotation magnitude; (d) Error δt in the translation magnitude.

realistic model of the functional axis of the joint, assuming that this functional axis is not varying too much in the range of movement between the two considered positions (i.e. the functional axis variation remains in the scale of calculation error).

In order to illustrate the interest of this model in a clinical context, we have chosen to show the improvement obtained in the determination of the flexion/extension axis of the elbow joint when distinct positions (i and j) are considered instead of successive positions (i and $i + 1$). To quantify the errors, the experiment has been realized on a fleshless upper-limb where the flexion/extension axis was marked by a stick crossing the elbow joint. This stick has been fixed by an anatomist and its position was defined by two reflective markers which was assumed to represent the reference functional axis. The humerus was fixed in the laboratory frame, and its position was located by four reflective markers. The radius and ulna were fastened together and this rigid body was also located by three markers. The forearm was moved in order to realize a flexion of the elbow joint to about 90° (see Fig. 4). The 3D trajectories of the reflective markers fixed on the humerus and the radius–ulna segments are recorded using a Motion Analysis system. From these trajectories, the relative displacement of the radius–ulna with respect to the humerus is calculated.

Fig. 5(a) and (b) display the errors on the direction (angle ε) and position (distance d) when the corresponding FHA is computed between two successive positions i and $i + 1$. Fig. 5(c) and (d). display the same errors when an amplitude of 45° is imposed between the two considered positions i and j. The curves show a real improvement in the accuracy in the second case (distinct positions); the maximum angular error is reduced from 25° to 3°, and the maximum linear error is reduced from about 4 cm to less than 1 cm. It has to be emphasized that the experimental conditions were dealing with an am-

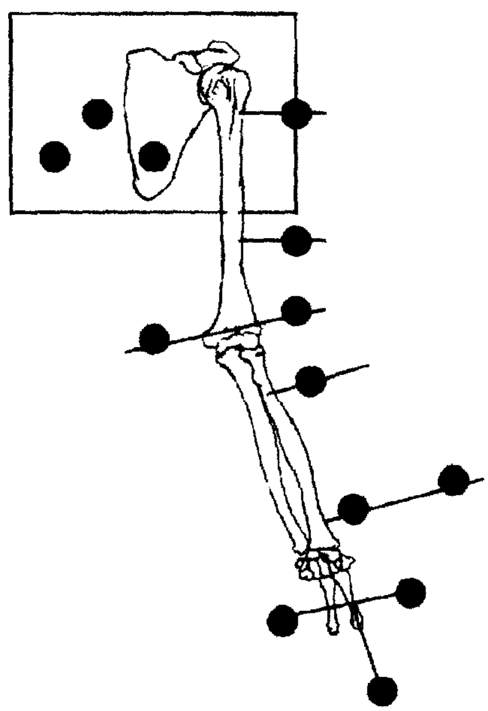


Fig. 4. Sketch representing the experimentation on the fleshless upper-limb. The scapula and the humerus are rigidly fastened on the base. The radius and the ulna are fastened together and each body segment (scapula, humerus, radius and hand) is located by at least three reflective markers mounted on sticks directly screwed in bones. The functional flexion/extension axis of the elbow is materialized by a stick located by two reflective markers.

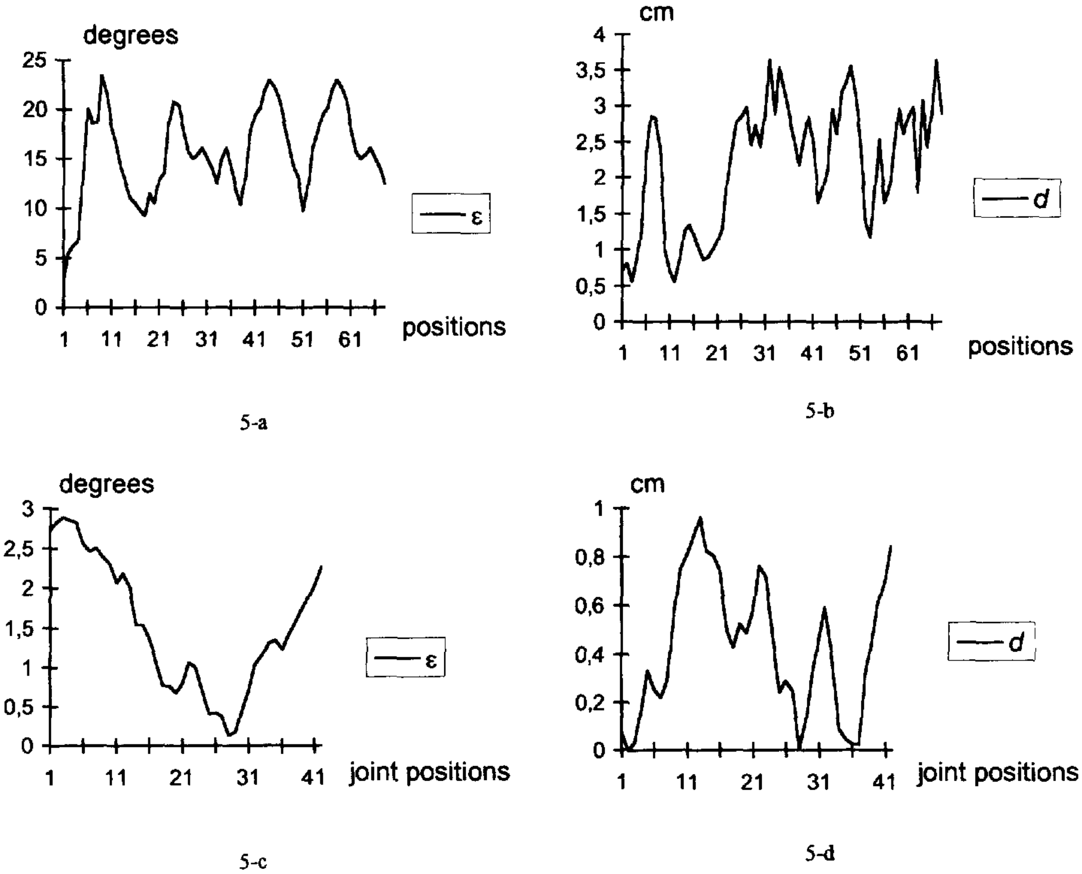


Fig. 5. Errors in the FHA characteristics corresponding to the experiment on the fleshless upper-limb: flexion of the elbow joint. (a) and (b) correspond to the calculation between successive position i, $i + 1$. (c) and (d) correspond to the calculation between distinct positions i and j. (a) Angle ε between the axis defined by the markers fixed on the humerus and the FHA computed between two successive positions i, $i + 1$; (b) Linear distance d between these two axes; (c) Angle ε between the axis defined by the markers fixed on the humerus and the FHA computed using a minimum of 45° of rotation between two considered positions; (d) Linear distance d between these two axes.

plitude of measurement error less than 1 cm, as the reflective markers were fixed directly on the bones so that the major source of error (i.e. the perturbations due to the muscular and adipose tissues displacements with respect to the skeleton) was avoided.

4. Conclusion

The finite helical displacement computed between two adjacent body segments in two successive positions is often used to approximate the instantaneous joint movement. A quantification of the errors on each FHA parameter has been realized on a numerical simulation of the knee joint evolution during gait. This case has been chosen because experimental measurements of the distances between skin-markers and corresponding anatomical landmarks were available.

The results obtained on this numerical simulation have pointed out that the determination of the finite helical axis characterizing the finite displacement of anatomical joints from noisy measurements is accurate only if the variation between the two considered positions is sufficient with respect to the perturbations. For example, in the case of an experiment using a video-based motion analysis system (maximum measurement error = 2 cm), the characterization of a displacement smaller than about 20° to 25° of rotation is not reasonable. Because of this limit, the FHA is not a good way to describe the variation of a joint displacement during a given movement.

It is much better, to obtain fair results, to model the joint kinematics by a sequence of rotations about three successive axes (Chao, 1980; Grood and Suntay, 1983). Nevertheless, this last description has an interest in a clinical context only if the chosen axes are consistent with the functional axes of the joint under study. These functional axes are very difficult to determine correctly, as Pennock and Clark (1990) and Ramakrishnan and Kadaba (1991) have shown, but they are the basis for a reliable clinical analysis of the movement. The approach proposed here, i.e. calculating a mean FHA between sufficiently distinct positions, is a good way to define the joint functional axis, provided this axis can be assumed steady between these two joint positions. The experimental results obtained on the fleshless upper-limb indicate that the FHA well represents the functional flexion/extension axis of the elbow joint, if the calculations are realized between enough distinct positions.

As a conclusion, the calculation of the finite helical axis between two sufficiently distinct positions is undoubtedly useful to determine accurately the joint functional axes from a recording of corresponding pure movements (i.e., flexion/extension, internal/external rotation) for each joint under study. The curves displayed in this paper allow the choice of the appropriate rotation increment between the two selected joint positions in order to obtain reliable results for data corresponding either to skin-fixed marker trajectories, or to anatomical landmarks digitalized on radiographs.

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